Timescales of eddy activity in the Southern Ocean

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Southern Ocean Momentum Balance

Existing Theoretical Framework (Gill et. al. 1974, Marshall & Speer, 2012; Hallberg & Gnanadesikan, 2006; Abernathey & Cessi, 2014): Competition between wind driven upwelling and baroclinic eddies determines mean isopycnal slope, ACC transport, and MOC.
Meredith & Hogg (2006, GRL)

Wind Stress $\rightarrow$ APE $\rightarrow$ EKE

2 ~ 3 yrs
Meredith & Hogg (2006, GRL)

Wind Stress $\rightarrow$ APE $\rightarrow$ EKE

Simple model: channel SO

Variable Wind forcing at surface

Adiabatic Interior
Meredith & Hogg (2006, GRL)

Wind Stress $\rightarrow$ APE $\rightarrow$ EKE

Simple model:
Energy pathway

\[
\frac{d(APE)}{dt} = W - C \\
\frac{d(EKE)}{dt} = C - D
\]
Meredith & Hogg (2006, GRL)

Wind Stress $\rightarrow$ APE $\rightarrow$ EKE

Simple model:
Energy pathway

\[
\frac{d(APE)}{dt} = W - C
\]

Conversion term

\[
\frac{d(EKE)}{dt} = C - D
\]

Drag term

Variable Wind forcing at surface

Simple model: channel SO

Adiabatic Interior

\[
C = - \int \int \int _{dV} \bar{w'}b'
\]

\[
\sim -K_{GM} \frac{(\nabla b)^2}{N^2}
\]
Simple model: w/o Eddy feedback

\[ \frac{dP(t)}{dt} = f(t) - cP(t) \]
\[ \frac{dK(t)}{dt} = cP(t) - rK(t) \]

GM type closure

linear bottom drag

Transfer Function

\[ f = \hat{f} e^{i\omega t} = \hat{f} e^{i\omega t + \phi_f} \]
\[ P = \hat{P} e^{i\omega t} = \hat{P} e^{i\omega t + \phi_P} \]
\[ K = \hat{K} e^{i\omega t} = \hat{K} e^{i\omega t + \phi_K} \]

\[ c \sim \frac{K_{GM}}{L_y^2} \approx 10^{-9} \text{ s}^{-1} \quad (\sim 3 \text{ years !}) \]
Simple model: w/o Eddy feedback

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linear bottom drag

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\[ c \sim \frac{K_{GM}}{L_y^2} \approx 10^{-9} \, s^{-1} \quad (\sim 3 \text{ years}) ! \]
Simple model: w/o Eddy feedback

\[ \hat{K} \left\| \frac{f}{\hat{f}} \right\|, \quad \hat{P} \left\| \frac{f}{\hat{f}} \right\| \]

Gain

Phase Angle

\[ \phi_K - \phi_f, \quad \phi_P - \phi_f \]
Simple model: w/o Eddy feedback

Low Freq. Limit
("eddy saturation")

- SMALL change in isopycnal slope (APE)
- BIG change in EKE

High Freq. Limit
(Ekman)

- BIG change in isopycnal slope (APE)
- SMALL change in EKE

\[ \hat{K}, \hat{P}, \hat{f} \]
Simple model

w/o Eddy feedback

\[
\frac{dP(t)}{dt} = f(t) - cP(t)
\]

\[
\frac{dK(t)}{dt} = cP(t) - rK(t)
\]

Sinha & Abernathey (submitted to JPO)
Simple model

w/o Eddy feedback

\[
\frac{dP(t)}{dt} = f(t) - cP(t)
\]

\[
\frac{dK(t)}{dt} = cP(t) - rK(t)
\]

with Eddy feedback

based on mixing length arguments

\[
\frac{dP}{dt} = f - kPK^\alpha
\]

\[
\frac{dK}{dt} = kPK^\alpha - r_1 K^{\beta}
\]

general bottom drag

linearize and solve for

\[
\frac{dP'}{dt} = f' - c_1 P' - c_2 K'
\]

\[
\frac{dK'}{dt} = c_1 P' + c_2 K' - r K'
\]

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Simple model

w/o Eddy feedback

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\frac{dP(t)}{dt} = f(t) - cP(t)
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Simple model

w/o Eddy feedback

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\frac{dP(t)}{dt} = f(t) - cP(t)
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\]

Sinha & Abernathey (submitted to JPO)

Two conversion terms:

- Same as c, (eddy mixing coeff)
- New term, (eddy feedback)

\[c_1, c_2\]
Isopycnal GOLD model:
(Hallberg & Gnanadesikan, 2001, 2006; Howard et al. 2015)

- Reduced gravity model
- 4 km horizontal resolution
- Three isopycnal layers
- Wind forcing only

Seven experiments
- Steady sinusoidal wind jet (0.2 N/m²)
- plus oscillations +/- (0.1 N/m²), 0.25, 0.5, 1, 2, 4, 8 year periods

Diagnostics
- EKE
- APE
- Wind Energy input

Sinha & Abernathey (submitted to JPO)
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Diagnostics
- EKE
- APE
- Wind Energy input

Sinha & Abernathey (submitted to JPO)
Spectral Analysis

energy input: same power, different frequency

response: different amplitudes

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Composite Analysis

$N_T$ forcing cycles

wind forcing cycle

forcing period $T$

$time series of each diagnostic for all ensemble members$

$M$ ensemble members

averaged over all ensemble members and all forcing cycles

composite over all forcing cycles and all ensemble members (T periodic signal)
Composite Analysis

90 day forcing

Eddy Kinetic Energy \( [m^2 s^{-2}] \)

Available Potential Energy \( [W m^{-2}] \)

Wind Power \( [W m^{-2}] \)

Time [days]

Sinha & Abernathey (submitted to JPO)
Composite Analysis

720 day forcing

Eddy Kinetic Energy \([m^2 s^{-2}]\)

Available Potential Energy \([m^2 s^{-2}]\)

Wind Power \([W m^{-2}]\)

Time [days]

Sinha & Abernathey (submitted to JPO)
Composite Analysis

2880 day forcing

Sinha & Abernathey (submitted to JPO)
Compare Analytic & Numerical Model

\[
\frac{dP'}{dt} = f' - c_1 P' - c_2 K' \\
\frac{dK'}{dt} = c_1 P' + c_2 K' - r K'
\]

**weak eddy feedback**

\[c_1 = \frac{f}{\overline{P}}; \quad c_2 = \frac{f}{2\overline{K}}\]

\[\sim 560 \text{ days} \quad \sim 157 \text{ days}\]

**strong eddy feedback**

\[c_1 = \frac{f}{\overline{P}}; \quad c_2 = \frac{f}{\overline{K}}\]

\[\sim 560 \text{ days} \quad \sim 78 \text{ days}\]

Sinha & Abernathey (submitted to JPO)
Summary

- Two limits: **Fast** vs **Slow** - Transient response to changing winds

- **Analytical model**: Energy Budget - wind power, APE, EKE
  - with and without eddy feedback

- Smooth **transfer function**, complex **phase** and **amplitude**
  - response to changing winds: **Regime shift**

- **Numerical simulations** with idealized model

- **Mechanistic description** of the eddy equilibration process with purely dynamic forcing
Discussions

- Eddy generation and dissipation - non-local in time

- Eddy memory effect - Time dependent eddy parameterization

- Used in conjunction with multiple timescale response to thermodynamic forcing (Ferreira et al. 2014) (sea ice, ozone depletion etc.) - more complete theory for SO response, baroclinic eddy equilibration
Thank you.
Simple model: Energy pathway

\[ \int \int_{dA} \tau \cdot u \]

\[ W \]

\[ K_m \]

\[ C_{K_m \rightarrow P_m} = W \]

\[ P_m \]

\[ C_{P_m \rightarrow P_t} \]

\[ P_t \]

\[ D \]

\[ K_t \]

\[ C_{P_t \rightarrow K_t} = C \]

\[ \text{wind work} \]

\[ \text{dissipation} \]

\[ \text{wind work} \]

\[ \text{dissipation} \]
Simple model: Energy pathway

\[ \int \int_{dA} \tau \cdot u \]

wind work

\[ W \]

\[ K_m \]

\[ C_{Km \rightarrow P_m} = W \]

\[ P_m \]

\[ C_{Pm \rightarrow P_t} \]

\[ P_t \]

dissipation

\[ D \]

\[ K_t \]

\[ C_{Pt \rightarrow K_t} = C \]

EKE

APE
Simple model: Energy pathway

\[ \int \int_{dA} \tau \cdot \mathbf{u} \]

\[ W \]

\[ K_m \]

\[ C_{K_m \rightarrow P_m} = W \]

\[ P_m \]

\[ C_{P_m \rightarrow P_t} \]

\[ P_t \]

\[ D \]

\[ K_t \]

\[ C_{P_t \rightarrow K_t} = C \]

\[ EKE \]

\[ APE \]

wind work

dissipation

steady-state balance
Simple model: Energy pathway

wind work

\[
\int \int_{dA} \tau \cdot u \quad \Rightarrow \quad W
\]

dissipation

\[
D
\]

steady-state balance

wind power input

\[
\frac{d(APE)}{dt} = W - C
\]

Conversion term

\[
\frac{d(EKE)}{dt} = C - D
\]

Drag term

\[
C_{Pm \rightarrow Pt} = APE
\]

\[
P_m \quad \Rightarrow \quad P_t
\]
Appendix

- **Meredith and Hogg (2006)** - EKE peaks around 2-3 years after SAM: Wind Energy stored as PE slowly transferred to EKE
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- **Wilson et al (2015)**: Energy Budget. Subannual timescales (PE -> KE) vs. annual/decadal (PE -> Friction)

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Appendix

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• But there is more than one timescale!
Appendix

- **Meredith and Hogg (2006)** - EKE peaks around 2-3 years after SAM: Wind Energy stored as PE slowly transferred to EKE

- **Wilson et al (2015)**: Energy Budget. Subannual timescales (PE -> KE) vs. annual/decadal (PE -> Friction)

\[
W + B_{ML} = \frac{d}{dt}(PE) + \frac{d}{dt}(KE) + D + V
\]

- But there is more than one timescale!
Appendix
Appendix

- Treguer et. al. (2010): response to increase in SAM in eddy permitting model
Appendix

- **Treguier et. al. (2010):** response to increase in SAM in eddy permitting model
Simple model with Eddy Feedback
Power Spectra of Winds

NCAR Reanalysis

ERA Interim
Spectral Amplitude Response

Gain (from spectra)

Forcing Period, [years]
Amplitude and Phase from Composite